Mathematics 1 Critical Areas of Focus

Ohio's Learning Standards for Mathematics include descriptions of the Conceptual Categories. These descriptions have been used to develop critical areas for each of the courses in both the Traditional and Integrated pathways. The critical areas are designed to bring focus to the standards in each course by describing the big ideas that educators can use to build their high school curriculum and to guide instruction. Each course contains up to six critical areas. This document identifies the clusters and standards that build toward each critical area.

The purpose of this document is to facilitate discussion among teachers and curriculum experts and to encourage coherence in the sequence, pacing and units of study for high school curriculum. Professional learning communities can use the following questions as examples to develop their high school curriculum.

DISCUSSION QUESTIONS

Example 1: Analyze and discuss the content for each high school course's Critical Areas of Focus.
- What are the concepts?
- What are the procedures and skills?
- What are the key mathematical practices?
- What are the relationships students are to make?
- What further information is needed? For example, what does prove mean?
- What are appropriate models for representing this learning?

Example 2: Identify and discuss the connections among the conceptual categories, domains, clusters and standards <u>within</u> each course's Critical Areas of Focus.
- What are the relationships among the conceptual categories, domains, clusters and standards?
- Why is each relationship important?
- What are the differences?
- How does the Critical Area of Focus description inform the instruction of the related conceptual categories, domains, clusters and standards?

Example 3: Identify and discuss any connections <u>across</u> the Critical Areas of Focus within a course. This information will help create a sequence of units for the course. For example, Critical Area of Focus #1 is needed prior to the learning in Critical Area of Focus #3.

Example 4: Compare each Critical Area of Focus to those for the preceding and succeeding courses to become familiar with previous and future learning.
- What understandings does this learning build upon?
- What are the related future understandings?

Example 5: Compare and contrast Ohio's Learning Standards to the current district curriculum.
- What is taught now but not in Ohio's Learning Standards?
- What content is essentially the same? Identify the differences.
- What will be new content for this grade?

Mathematics 1 Table of Contents

Mathematics 1 Critical Areas of Focus

CRITICAL AREA OF FOCUS #1

Relationships Between Quantities

By the end of eighth grade students have learned to solve linear equations in one variable. Now students build on these earlier experiences to develop fluency writing, interpreting, and translating between various forms of linear and simple exponential expressions. They master the solution of linear equations and inequalities to apply related solution techniques and the laws of exponents in the creation and solution of simple exponential equations. All this work is grounded on understanding quantities and on the relationships between them. Students apply this learning in real-world and modeling situations.

Number and Quantity - Quantities

Reason quantitatively and use units to solve problems.

N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★

N.Q.2 Define appropriate quantities for the purpose of descriptive modeling. ★

N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. ★

Algebra – Seeing Structure in Expressions

Interpret the structure of expressions.

A.SSE.1 Interpret expressions that represent a quantity in terms of its context. ★

- **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
- **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity.

A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★

c. Use the properties of exponents to transform expressions f

Algebra – Creating Equations

Create equations that describe numbers or relationships.

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.* ★

- **a.** Focus on applying linear and simple exponential expressions. (A1, M1)

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★

- **a.** Focus on applying linear and simple exponential expressions. (A1, M1)

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. ★ (A1, M1)

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. ★

- **b.** Focus on formulas in which the variable of interest is linear. *For example, rearrange Ohm's law V = IR to highlight resistance R.* (M1)

CRITICAL AREA OF FOCUS #2

Linear and Exponential Relationships

In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. Students will learn function notation and develop the concepts of domain and range. Their understanding moves beyond viewing functions as processes that take inputs and yield outputs and to viewing functions as objects in their own right followed by an informal introduction of inverse functions. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate or incomplete. Their work includes functions that can be described or modeled by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Algebra – Reasoning with Equations and Inequalities

Represent and solve equations and inequalities graphically.

A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A.REI.11 Explain why the x-coordinates of the points where the graphs of the equation $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.

A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Functions – Interpreting Functions

Understand the concept of a function, and use function notation.

F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for $n \geq 1$.*

Interpret functions that arise in applications in terms of the context.

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★(A2, M3)

a. Focus on linear and exponential functions. (M1)

Critical Area of Focus #2 CONTINUED

Linear and Exponential Relationships

Functions – Interpreting Functions

Interpret functions that arise in applications in terms of the context.

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* ★

- **a.** Focus on linear and exponential functions. (M1)

Analyze functions using different representations.

F.IF.7 Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. ★

- **a.** Graph linear functions and indicate intercepts. (A1, M1)
- **e.** Graph simple exponential functions, indicating intercepts and end behavior. (A1, M1)

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (A2, M3)

- **a.** Focus on linear and exponential functions. (M1)

Functions – Building Functions

Build a function that models a relationship between two quantities.

F.BF.1 Write a function that describes a relationship between two quantities. ★

- **a.** Determine an explicit expression, a recursive process, or steps for calculation from context.
  - **i.** Focus on linear and exponential functions. (A1, M1)

F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Build new functions from existing functions.

F.BF.4 Find inverse functions.

- **a.** Informally determine the input of a function when the output is known. (A1, M1)

Functions – Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models, and solve problems.

F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. ★

- **a.** Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
- **b.** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- **c.** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Critical Area of Focus #2 CONTINUED

Linear and Exponential Relationships

Functions – Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models, and solve problems.

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★

Interpret expressions for functions in terms of the situation they model.

F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context. ★

CRITICAL AREA OF FOCUS #3

Reasoning with Equations

By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical methods to analyze and solve systems of linear equations in two variables. This learning builds on these earlier experiences by asking students to analyze and explain the process of solving equations and inequalities to justify the process used in solving a system of equations. Students explore systems of equations and inequalities to find and interpret their solutions. All this work is grounded on understanding quantities and on the relationships between them. Students apply this learning in real-world situations.

Algebra – Reasoning with Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning.

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Solve systems of equations.

A.REI.5 Verify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A.REI.6 Solve systems of linear equations algebraically and graphically.

a. Limit to pairs of linear equations in two variables. (A1, M1)

Mathematics 1 Critical Areas of Focus

CRITICAL AREA OF FOCUS #4

Descriptive Statistics

In middle school, students developed an understanding of statistical problem solving through the format of the GAISE Model. They were expected to display numerical data and summarize it using measures of center and variability. By the end of middle school, students were creating scatterplots and recognizing linear trends in data. Now, they apply those concepts by using the GAISE model in the context of real-world applications. Students develop formal means of assessing how a model fits data. They use regression techniques to describe approximately linear relationships between quantities. Students use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. In Algebra 2/Mathematics 3, students will look at residuals to analyze the goodness of fit.

Statistics and Probability

Summarize, represent, and interpret data on a single count or measurement variable

S.ID.1 Represent data with plots on the real number line (dot plots[G], histograms, and box plots) in the context of real-world applications using the GAISE model.★

S.ID.2 In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviation, interquartile range, and standard deviation) of two or more different data sets. ★

S.ID.3 In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★

Summarize, represent, and interpret data on two categorical and quantitative variables.

S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in The data. ★

S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. ★

c. Fit a linear function for a scatterplot that suggests a linear association. (A1, M1)

Interpret linear models.

S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. ★

S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit. ★

Mathematics 1 Critical Areas of Focus

CRITICAL AREA OF FOCUS #5

Congruence, Proof, and Constructions

In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections, and rotations and have used these to develop notions about what it means for two objects to be congruent or to have symmetries of itself, rotational or reflected. Students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They use triangle congruence as a familiar foundation for the development of formal and informal proof. Students prove theorems—using a variety of formats—and apply them when solving problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete geometric constructions and explain why they work. Students will extend prior experience with geometric shapes toward the development of a hierarchy of two-dimensional figures based on formal properties.

Geometry - Congruence

Experiment with transformations in the plane.

G.CO.1 Know precise definitions of ray, angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and arc length.

G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, e.g., translation versus horizontal stretch.

G.CO.3 Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself.

- **a.** Identify figures that have line symmetry; draw and use lines of symmetry to analyze properties of shapes.
- **b.** Identify figures that have rotational symmetry; determine the angle of rotation, and use rotational symmetry to analyze properties of shapes.

G.CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using items such as graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions.

G.CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

G.CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

G.CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems both formally and informally using a variety of methods.

G.CO.9 Prove and apply theorems about lines and angles. *Theorems include but are not restricted to the following: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*

CRITICAL AREA OF FOCUS #5 CONTINUED

Congruence, Proof, and Constructions

Geometry - Congruence

Prove geometric theorems both formally and informally using a variety of methods.

G.CO.10 Prove and apply theorems about triangles. *Theorems include but are not restricted to the following: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

G.CO.11 Prove and apply theorems about parallelograms. *Theorems include but are not restricted to the following: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Make geometric constructions.

G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

G.CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Classify and analyze geometric figures.

G.CO.14 Classify two-dimensional figures in a hierarchy based on properties.

Mathematics 1 Critical Areas of Focus

CRITICAL AREA OF FOCUS #6

Connecting Algebra and Geometry Through Coordinates

Building on their work with the Pythagorean Theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of triangles and quadrilaterals and slopes of parallel and perpendicular lines.

Geometry – Expressing Geometric Properties with Equations

Use coordinates to prove simple geometric theorems algebraically and to verify specific geometric statements.

G.GPE.5 Justify the slope criteria for parallel and perpendicular lines, and use them to solve geometric problems, e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point.

G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. ★

CRITICAL AREA OF FOCUS #7

Circles

Students prove and apply basic theorems about circles dealing with lines tangent to a circle, inscribed and central angles, chords and secants. They study relationships among segments on chords, secants, and tangents as an application of similarity.

Geometry – Circles

Understand and apply theorems about circles.

G.C.2 Identify and describe relationships among angles, radii, chords, tangents, and arcs and use them to solve problems. *Include the relationship between central, inscribed, and circumscribed angles and their intercepted arcs; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*

G.C.3 Construct the inscribed and circumscribed circles of a triangle; prove and apply the property that opposite angles are supplementary for a quadrilateral inscribed in a circle.

(+) **G.C.4** Construct a tangent line from a point outside a given circle to the circle.