Ohio's Learning Standards | Mathematics

Grade 8

Ohio | Department of Education

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Introduction

PROCESS

To better prepare students for college and careers, educators used public comments along with their professional expertise and experience to revise Ohio's Learning Standards. In spring 2016, the public gave feedback on the standards through an online survey. Advisory committee members, representing various Ohio education associations, reviewed all survey feedback and identified needed changes to the standards. Then they sent their directives to working groups of educators who proposed the actual revisions to the standards. The Ohio Department of Education sent their revisions back out for public comment in July 2016. Once again, the Advisory Committee reviewed the public comments and directed the Working Group to make further revisions. Upon finishing their work, the department presented the revisions to the Senate and House education committees as well as the State Board of Education.

UNDERSTANDING MATHEMATICS

These standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true, or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic device comes from. The student who can explain the rule understands the mathematics at a much deeper level. Then the student may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The content standards are grade-specific. However, they do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the standards to define the full range of supports appropriate for English learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. Educators should read the standards allowing for the widest possible range of students to participate fully from the outset. They should provide appropriate accommodations to ensure maximum participation of students with special education needs. For example, schools should allow students with disabilities in reading to use Braille, screen reader technology or other assistive devices. Those with disabilities in writing should have scribes, computers, or speech-to-text technology. In a similar vein, educators should interpret the speaking and listening standards broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the standards do provide clear signposts along the way to help all students achieve the goal of college and career readiness.

The standards begin on page 4 with the eight Standards for Mathematical Practice.

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up:* adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.
In grade 8, students solve real-world problems through the application of algebraic and geometric concepts. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?"

2. Reason abstractly and quantitatively.
In grade 8, students represent a wide variety of real-world contexts through the use of real numbers and variables in mathematical expressions, equations, and inequalities. They examine patterns in data and assess the degree of linearity of functions. Students contextualize to understand the meaning of the number(s) or variable(s) as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.

3. Construct viable arguments and critique the reasoning of others.
In grade 8, students construct arguments using verbal or written explanations accompanied by expressions, equations, inequalities, models, and graphs, tables, and other data displays (i.e. box plots, dot plots, histograms, etc.). They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They pose questions like "How did you get that?", "Why is that true?" "Does that always work?" They explain their thinking to others and respond to others' thinking.

4. Model with mathematics.
In grade 8, students model problem situations symbolically, graphically, in tables, and contextually. Working with the new concept of a function, students learn that relationships between variable quantities in the real-world often satisfy a dependent relationship, in that one quantity determines the value of another. Students form expressions, equations, or inequalities from real-world contexts and connect symbolic and graphical representations. Students use scatterplots to represent data and describe associations between variables. Students need many opportunities to connect and explain the connections between the different representations. They should be able to use all of these representations as appropriate to a problem context. Students should be encouraged to answer questions such as "What are some ways to represent the quantities?" or "How might it help to create a table, chart, graph, or ___?"

5. Use appropriate tools strategically.
Students consider available tools (including estimation and technology) when solving a mathematical problem and decide when certain tools might be helpful. For instance, students in grade 8 may translate a set of data given in tabular form to a graphical representation to compare it to another data set. Students might draw pictures, use applets, or write equations to show the between the angles created by a transversal that intersects parallel lines. Teachers might ask, "What approach are you considering?" or "Why was it helpful to use ___?"

Standards for Mathematical Practice, continued

6. Attend to precision.

In grade 8, students continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to the number system, functions, geometric figures, and data displays. Teachers might ask, "What mathematical language, definitions, or properties can you use to explain ____?"

7. Look for and make use of structure.

Students routinely seek patterns or structures to model and solve problems. In grade 8, students apply properties to generate equivalent expressions and solve equations. Students examine patterns in tables and graphs to generate equations and describe relationships. Additionally, students experimentally verify the effects of transformations and describe them in terms of congruence and similarity.

8. Look for and express regularity in repeated reasoning.

In grade eight, students use repeated reasoning to understand the slope formula and to make sense of rational and irrational numbers. Through multiple opportunities to model linear relationships, they notice that the slope of the graph of the linear relationship and the rate of change of the associated function are the same. For example, as students repeatedly check whether points are on the line with a slope of 3 that goes through the point (1, 2), they might abstract the equation of the line in the form $\frac{y-2}{x-1} = 3$. Students should be encouraged to answer questions such as "How would we prove that ___?" or "How is this situation like and different from other situations using these operations?"

CONNECTING THE STANDARDS FOR MATHEMATICAL PRACTICE TO THE STANDARDS FOR MATHEMATICAL CONTENT

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

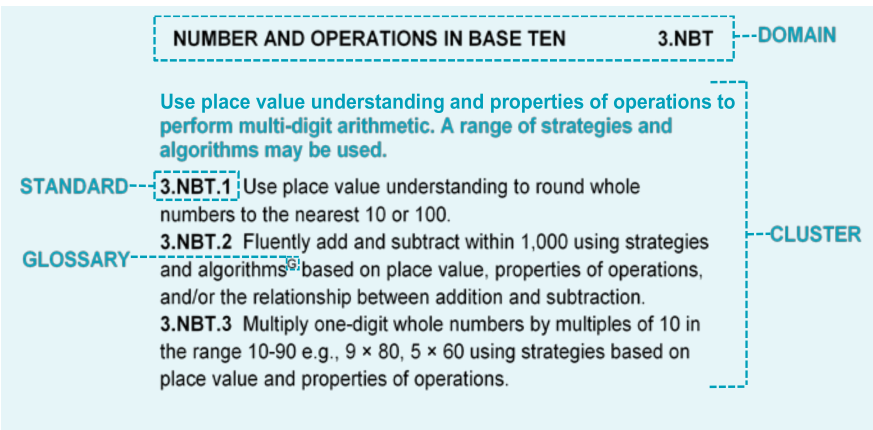
How to Read the Grade Level Standards

Standards define what students should understand and be able to do.

Clusters are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.

G shows there is a definition in the glossary for this term.



These standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, does not necessarily mean that teachers must teach topic A before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, "Students who already know ... should next come to learn ...." But at present this approach is unrealistic—not least because existing education research cannot specify all such learning pathways. Therefore, educators, researchers, and mathematicians used their collective experience and professional judgment along with state and international comparisons as a basis to make grade placements for specific topics.

Grade 8

In Grade 8, instructional time should focus on four critical areas:

Critical Area 1: Formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations
Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($^{y}/_{x} = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables graphically or by simple inspection; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

Critical Area 2: Grasping the concept of a function and using functions to describe quantitative relationships
Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

Critical Area 3: Analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem
Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Critical Area 4: Working with irrational numbers, integer exponents, and scientific notation
Students explore irrational numbers and their approximations. They extend work with expressions and equations with integer exponents, square and cube roots. Understandings of very large and very small numbers, the place value system, and exponents are combined in representations and computations with scientific notation.

GRADE 8 OVERVIEW

THE NUMBER SYSTEM

- Know that there are numbers that are not rational, and approximate them by rational numbers.

EXPRESSIONS AND EQUATIONS

- Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.

FUNCTIONS

- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities.

GEOMETRY

- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem.
- Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

STATISTICS AND PROBABILITY

- Investigate patterns of association in bivariate data.

MATHEMATICAL PRACTICES

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 8

THE NUMBER SYSTEM **8.NS**

Know that there are numbers that are not rational, and approximate them by rational numbers.

8.NS.1 Know that real numbers are either rational or irrational. Understand informally that every number has a decimal expansion which is repeating, terminating, or is non-repeating and non-terminating.

8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions, e.g., π^2. *For example, by truncating the decimal expansion of* $\sqrt{2}$*, show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

EXPRESSIONS AND EQUATIONS **8.EE**

Work with radicals and integer exponents.

8.EE.1 Understand, explain, and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8*; and the population of the world as* 7×10^9*; and determine that the world population is more than 20 times larger.*

8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal notation and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities, e.g., use millimeters per year for seafloor spreading. Interpret scientific notation that has been generated by technology.

Understand the connections between proportional relationships, lines, and linear equations.

8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Analyze and solve linear equations and pairs of simultaneous linear equations.

8.EE.7 Solve linear equations in one variable.

- **a.** Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
- **b.** Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

EXPRESSIONS AND EQUATIONS, continued

Analyze and solve linear equations and pairs of simultaneous linear equations. (continued)

8.EE.8 Analyze and solve pairs of simultaneous linear equations graphically.

- **a.** Understand that the solution to a pair of linear equations in two variables corresponds to the point(s) of intersection of their graphs, because the point(s) of intersection satisfy both equations simultaneously.
- **b.** Use graphs to find or estimate the solution to a pair of two simultaneous linear equations in two variables. Equations should include all three solution types: one solution, no solution, and infinitely many solutions. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*
- **c.** Solve real-world and mathematical problems leading to pairs of linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.* (Limit solutions to those that can be addressed by graphing.)

FUNCTIONS 8.F

Define, evaluate, and compare functions.

8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. Function notation is not required in Grade 8.

8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Use functions to model relationships between quantities.

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph, e.g., where the function is increasing or decreasing, linear or nonlinear. Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

GEOMETRY 8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.

8.G.1 Verify experimentally the properties of rotations, reflections, and translations (include examples both with and without coordinates).

- **a.** Lines are taken to lines, and line segments are taken to line segments of the same length.
- **b.** Angles are taken to angles of the same measure.
- **c.** Parallel lines are taken to parallel lines.

GEOMETRY, continued

Understand congruence and similarity using physical models, transparencies, or geometry software. (continued)

8.G.2 Understand that a two-dimensional figure is congruent[G] to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (Include examples both with and without coordinates.)

8.G.3 Describe the effect of dilations[G], translations, rotations, and reflections on two-dimensional figures using coordinates.

8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. (Include examples both with and without coordinates.)

8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Understand and apply the Pythagorean Theorem.

8.G.6 Analyze and justify an informal proof of the Pythagorean Theorem and its converse.

8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

8.G.9 Solve real-world and mathematical problems involving volumes of cones, cylinders, and spheres.

STATISTICS AND PROBABILITY 8.SP

Investigate patterns of association in bivariate data.

8.SP.1 Construct and interpret scatter plots for bivariate[G] measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering; outliers; positive, negative, or no association; and linear association and nonlinear association. (GAISE Model, steps 3 and 4)

8.SP.2 Understand that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. (GAISE Model, steps 3 and 4)

8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* (GAISE Model, steps 3 and 4)

8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

Glossary

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: $8 + 2 = 10$ is an addition within 10, $14 - 5 = 9$ is a subtraction within 20, and $55 - 18 = 37$ is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: $\frac{3}{4}$ and $-\frac{3}{4}$ are additive inverses of one another because $\frac{3}{4} + (-\frac{3}{4}) = (-\frac{3}{4}) + \frac{3}{4} = 0$.

Algorithm. *See also:* computation algorithm.

Associative property of addition. See Table 3, page 18.

Associative property of multiplication. See Table 3, page 18.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1] *See also:* first quartile and third quartile.

Commutative property. See Table 3, page 18.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Dot plot. *See also:* line plot.

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[1] Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

[2] Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," Journal of Statistics Education Volume 14, Number 3 (2006).

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.2 *See also:* median, third quartile, interquartile range.

Fluency. The ability to use efficient, accurate, and flexible methods for computing. Fluency does not imply timed tests.

Fluently. *See also:* fluency.

Fraction. A number expressible in the form $^a/_b$ where *a* is a whole number and *b* is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) *See also:* rational number.

Identity property of 0. See Table 3, page 18.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

Integer. A number expressible in the form *a* or −*a* for some whole number *a*.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 − 6 = 9. *See also:* first quartile, third quartile.

Justify: To provide a convincing argument for the truth of a statement to a particular audience.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list. (To be more precise, this defines the arithmetic mean) Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

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[3] Adapted from Wisconsin Department of Public Instruction, *op. cit.*

[4] Adapted from Wisconsin Department of Public Instruction, *op. cit.*

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: $72 \div 8 = 9$.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: $^3/_4$ and $^4/_3$ are multiplicative inverses of one another because $^3/_4 \times {}^4/_3 = {}^4/_3 \times {}^3/_4 = 1$.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by $^5/_{50}$ = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of equality. See Table 4, page 96.

Properties of inequality. See Table 5, page 97.

Properties of operations. See Table 3, page 18.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Prove: To provide a logical argument that demonstrates the truth of a statement. A proof is typically composed of a series of justifications, which are often single sentences, and may be presented informally or formally.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a nonzero denominator.

Rational number. A number expressible in the form $^{a}/_{b}$ or $-^{a}/_{b}$ for some fraction $^{a}/_{b}$. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

Similarity transformation. A rigid motion followed by a dilation.

Standard Algorithm. See computational algorithm.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median *M*, the third quartile is the median of the data values greater than *M*. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Trapezoid. 1. A trapezoid is a quadrilateral with at least one pair of parallel sides. (inclusive definition) 2. A trapezoid is a quadrilateral with exactly one pair of parallel sides. (exclusive definition) *Districts may choose either definition to use for instruction. Ohio's State Tests' items will be written so that either definition will be acceptable.*

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Verify: To check the truth or correctness of a statement in specific cases.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, ….

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[5] Adapted from Wisconsin Department of Public Instruction, *op. cit.*

Table 1. Common Addition and Subtraction Situations.

	RESULT UNKNOWN	CHANGE UNKNOWN	START UNKNOWN
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	TOTAL UNKNOWN	**ADDEND UNKNOWN**	**BOTH ADDENDS UNKNOWN[1]**
PUT TOGETHER/ TAKE APART[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	DIFFERENCE UNKNOWN	**BIGGER UNKNOWN**	**SMALLER UNKNOWN**
COMPARE[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1] These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean "makes" or "results in" but always does mean "is the same number as."

[2] Either addend can be unknown, so there are three variations of these problem situations. *Both Addends Unknown* is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

[3] For the *Bigger Unknown* or *Smaller Unknown* situations, one version directs the correct operation (the version using more for the *Bigger Unknown* and using less for the *Smaller Unknown*). The other versions are more difficult.

Table 2. Common Multiplication and Division Situations[1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 X 6 = ?	3 X ? = 18, AND 18 ÷ 3 = ?	? X 6 = 18, AND 18 ÷ 6 = ?
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? **Measurement example.** You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? **Measurement example.** You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? **Measurement example.** You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS[2], AREA[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? **Area example.** What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? **Area example.** A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? **Area example.** A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? **Measurement example.** A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? **Measurement example.** A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? **Measurement example.** A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[1] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

[2] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[3] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Table 3. Properties of Operations.

Here *a, b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system and the complex number sytem.

ASSOCIATIVE PROPERTY OF ADDITION	$(a+b)+c=a+(b+c)$
COMMUTATIVE PROPERTY OF ADDITION	$a+b=b+a$
ADDITIVE IDENTITY PROPERTY OF 0	$a+0=0+a=a$
EXISTENCE OF ADDITIVE INVERSES	For ever a there exists $-a$ so that $a+(-a)=(-a)+a=0$
ASSOCIATIVE PROPERTY OF MULTIPLICATION	$(a\times b)\times c=a\times(b\times c)$
COMMUTATIVE PROPERTY OF MULTIPLICATION	$a\times b=b\times a$
MULTIPLICATIVE IDENTITY PROPERTY OF 1	$a\times 1=1\times a=a$
EXISTENCE OF MULTIPLICATIVE INVERSES	For every $a\neq 0$ there exists $\frac{1}{a}$ so that $a\times\frac{1}{a}=\frac{1}{a}\times a=1$
DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER ADDITION	$a\times(b+c)=a\times b+a\times c$

Table 4. Properties of Equality.

Here *a, b* and *c* stand for arbitrary numbers in the rational, real or complex number sytems.

REFLEXIVE PROPERTY OF EQUALITY	$a=a$
SYMMETRIC PROPERTY OF EQUALITY	If $a=b$, then $b=a$.
TRANSITIVE PROPERTY OF EQUALITY	If $a=b$ and $b=c$, then $a=c$.
ADDITION PROPERTY OF EQUALITY	If $a=b$, then $a+c=b+c$.
SUBTRACTION PROPERTY OF EQUALITY	If $a=b$, then $a-c=b-c$.
MULTIPLICATION PROPERTY OF EQUALITY	If $a=b$, then $a\times c=b\times c$.
DIVISION PROPERTY OF EQUALITY	If $a=b$ and $c\neq 0$, then $a\div c=b\div c$.
SUBSTITUTION PROPERTY OF EQUALITY	If $a=a$, then b may be substituted for a in any expression containing a.

Table 5. Properties of Inequality.

Here *a*, *b* and *c* stand for arbitrary numbers in the rational or real number sytems.

Exactly one of the following is true: $a < b, a = b, a > b$.
If $a > b$ and $b > c$, then $a > c$.
If $a > b$, then $b < a$.
If $a > b$, then $-a < -b$.
If $a > b$, then $a \pm c > b \pm c$.
If $a > b$ and $c > 0$, then $a \times c > b \times c$.
If $a > b$ and $c < 0$, then $a \times c < b \times c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Acknowledgements

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