Introduction to Compacted Standards in Middle School

HIGH SCHOOL MATHEMATICS IN MIDDLE SCHOOL[1]

There are some students who are able to move through the mathematics quickly. These students may choose to take high school mathematics beginning in eighth grade[2] or earlier, so they can take college-level mathematics in high school.[3] Students who are capable of moving more quickly deserve thoughtful attention, both to ensure that they are challenged and that they are mastering the full range of mathematical content and skills—without omitting critical concepts and topics. Care must be taken to ensure that students master and fully understand all important topics in the mathematics curriculum, and that the continuity of the mathematics learning progression is not disrupted. In particular, the Standards for Mathematical Practice ought to continue to be emphasized in these cases.

The number of students taking high school mathematics in eighth grade has increased steadily for years. Part of this trend is the result of a concerted effort to get more students to take Calculus and other college-level mathematics courses in high school. Enrollment in both AP Statistics and AP Calculus, for example, have essentially doubled over the last decade (College Board, 2009). There is also powerful research showing that among academic factors, the strongest predictor of whether a student will earn a bachelor's degree is the highest level of mathematics taken in high school (Adelman, 1999). A recent study completed by The College Board confirms this. Using data from 65,000 students enrolled in 110 colleges, students' high school coursework was evaluated to determine which courses were closely associated with students' successful performance in college. The study confirmed the importance of a rigorous curriculum throughout a students' high school career. Among other conclusions, the study found that students who took more advanced courses, such as Pre-Calculus in the 11th grade or Calculus in 12th grade, were more successful in college. Students who took AP Calculus at any time during their high school careers were most successful (Wyatt & Wiley, 2010). And even as more students are enrolled in more demanding courses, it does not necessarily follow that there must be a corresponding decrease in engagement and success (Cooney & Bottoms, 2009, p. 2).

At the same time, there are cautionary tales of pushing underprepared students into the first course of high school mathematics in the eighth grade. The Brookings Institute's 2009 Brown Center Report on American Education found that the NAEP scores of students taking Algebra 1 in the eighth grade varied widely, with the bottom ten percent scoring far below grade level. And a report from the Southern Regional Education Board, which supports increasing the number of middle students taking Algebra 1, found that among students in the lowest quartile on achievement tests, those enrolled in higher-level mathematics had a slightly higher failure rate than those enrolled in lower-level mathematics (Cooney & Bottoms, 2009, p. 2). In all other quartiles, students scoring similarly on achievement tests were less likely to fail if they were enrolled in more demanding courses. These two reports are reminders that, rather than skipping or rushing through content, students should have appropriate progressions of foundational content to maximize their likelihoods of success in high school mathematics.

It is also important to note that notions of what constitutes a course called "Algebra 1" or "Math 1" vary widely. In Ohio's Learning Standards for Mathematics, students begin preparing for algebra in Kindergarten, as they start learning about the properties of operations. Furthermore, much of the content central to typical Algebra 1 courses—namely linear equations, inequalities, and functions—is found in the 8th grade Learning Standards for Mathematics. The Algebra 1 course described here ("High School Algebra 1"), however, is the first formal algebra course in the Traditional Pathway (concepts from this Algebra 1 course are developed across the first two courses of the integrated pathway). Enrolling an eighth-grade student in a watered down version of either

[1] This section refers to mathematics content, not high school credit. The determination for high school credit is presumed to be made by state and local education agencies

[2] Either 8th Grade Algebra 1 or 8th Grade Mathematics 1

[3] Such as Calculus or Advanced Statistics.

the Algebra 1 course or Math 1 course described here may in fact do students a disservice, as mastery of algebra including attention to the Standards for Mathematical Practice is fundamental for success in further mathematics and on college entrance examinations. As mentioned above, skipping material to get students to a particular point in the curriculum will likely create gaps in the students' mathematical background, which may create additional problems later, because students may be denied the opportunity for a rigorous Algebra 1 or Math 1 course and may miss important content from eighth-grade mathematics.

COMPACTING HIGH SCHOOL AND MIDDLE SCHOOL STANDARDS

Taking the above considerations into account, as well as the recognition that there are other methods for accomplishing these goals, the Achieve Pathways Group endorses the notion that all students who are ready for rigorous high school mathematics in eighth grade should take such courses (Algebra 1 or Mathematics 1), and that all middle schools should offer this opportunity to their students. To prepare students for high school mathematics in eighth grade, districts are encouraged to have a well-crafted sequence of **compacted courses**. The term "compacted" means to compress content, which requires a faster pace to complete, as opposed to skipping content. The Achieve Pathways Group has developed two compacted course sequences, one designed for districts using a traditional Algebra 1 – Geometry – Algebra 2 high school sequence, and the other for districts using an integrated sequence, which is commonly found internationally. Both are based on the idea that content should compact 3 years of content into 2 years, at most. In other words, compacting content from 2 years into 1 year would be too challenging and compacting 4 years of content into 3 years starting in grade 7 runs the risk of compacting across middle and high schools. As such, grades 7, 8, and 9 were compacted into grades 7 and 8 (a 3:2 compaction). As a result, some 8th grade content is in the 7th grade courses, and high school content is in 8th grade.

The compacted traditional sequence, compacts grades 7, 8, and High School Algebra 1 into two years: "Compacted 7th Grade" and "8th Grade Algebra 1." Upon successful completion of this pathway, students will be ready for Geometry in high school. The compacted integrated sequence compacts grades 7, 8, and Math 1 into two years: "Compacted 7th Grade" and "8th Grade Mathematics 1." At the end of 8th grade, these students will be ready for Math 2 in high school. While the K-7 Learning Standards for Mathematics effectively prepare students for algebra in 8th grade, some standards from 8th grade have been placed in the Compacted 7th grade course to make the 8th grade courses more manageable.

The Achieve Pathways Group has followed a set of guidelines[4] for the development of these compacted courses.

1. **Compacted courses should include the same Ohio Learning Standards for Mathematics as the non-compacted courses.** It is recommended to compact three years of material into two years, rather than compacting two years into one. The rationale is that mathematical concepts are likely to be omitted when trying to squeeze two years of material into one. This is to be avoided, as the standards have been carefully developed to define clear learning progressions through the major mathematical domains. Moreover, the compacted courses should not sacrifice attention to the Mathematical Practices Standard.
2. **Decisions to accelerate students into the high school Learning Standards for Mathematics before ninth grade should not be rushed.** Placing students into tracks too early should be avoided at all costs. It is not recommended to compact the standards before grade seven. In this document, compaction begins in seventh grade for both the traditional and integrated (international) sequences.
3. **Decisions to accelerate students into high school mathematics before ninth grade should be based on solid evidence of student learning**. Research has shown discrepancies in the placement of students into "advanced" classes by race/ethnicity and socioeconomic background. While such decisions to accelerate are almost always a joint decision between the school and the family, serious

[4] Based on work published by Washington Office of the Superintendent of Public Schools, 2008

efforts must be made to consider solid evidence of student learning in order to avoid unwittingly disadvantaging the opportunities of particular groups of students.

4. **A menu of challenging options should be available for students after their third year of mathematics—and all students should be strongly encouraged to take mathematics in all years of high school.** Traditionally, students taking high school mathematics in the eighth grade are expected to take Precalculus in their junior years and then Calculus in their senior years. This is a good and worthy goal, but it should not be the only option for students. Advanced courses could also include Statistics, Discrete Mathematics, or Mathematical Decision Making. An array of challenging options will keep mathematics relevant for students, and give them a new set of tools for their futures in college and career (see Fourth Courses section of this paper for further detail).

OTHER WAYS TO ACCELERATE STUDENTS

Just as care should be taken not to rush the decision to accelerate students, care should also be taken to provide more than one opportunity for acceleration. Some students may not have the preparation to enter a "Compacted Pathway" but may still develop an interest in taking advanced mathematics, such as AP Calculus or AP Statistics in their senior year. Additional opportunities for acceleration may include the following:

- Allowing students to take two mathematics courses simultaneously (such as Geometry and Algebra 2, or Precalculus and Statistics).
- Allowing students in schools with block scheduling to take a mathematics course in both semesters of the same academic year.
- Offering summer courses that are designed to provide the equivalent experience of a full course in all regards, including attention to the Mathematical Practices.[5]
- Creating different compaction ratios, including four years of high school content into three years beginning in 9th grade.
- Creating a hybrid Algebra 2-Precalculus course that allows students to go straight to Calculus. A combination of these methods and our suggested compacted sequences would allow for the most mathematically-inclined students to take advanced mathematics courses during their high school career.

[5] As with other methods of accelerating students, enrolling students in summer courses should be handled with care, as the pace of the courses likely be enormously fast.

COMPACTED TRADITIONAL SEQUENCE

A suggested "compacted" version of the Traditional pathway is given where no content is omitted, in which students would complete the content of 7th grade, 8th grade, and the High School Algebra 1 course in grades 7 (Compacted 7th Grade) and 8 (8th Grade Algebra 1), which will enable them to reach Calculus or other college level courses by their senior year. While the K-7 Ohio Learning Standards effectively prepare students for algebra in 8th grade, some standards from 8th grade have been placed in the Compacted 7th Grade course to make the 8th Grade Algebra 1 course more manageable.

Topic	Year 1			Year 2	
Description	(Grade 7, part of grade 8, equations and functions of Algebra 1)			(Part of grade 8 and the rest Algebra 1)	
Test	Grade 7 Test			Algebra 1 Test	
Ratio and Proportions	7.RP.1	8.EE.5			
	7.RP.2	8.EE.6			
	7.RP.3				
	7.G.1				
Integer Operations	7.NS.1				A.APR.1a
	7.NS.2				
	7.NS.3				
Linear Expressions, Equations, and Inequalities	7.EE.1	8.EE.7	A.SSE.1 (linear)		A.CED.3
	7.EE.2	8.EE.8	A.APR.1 (linear)		A.CED.4
	7.EE.3		A.CED.1a (linear)		A.REI.5
	7.EE.4		A.CED.2a (linear)		A.REI.6
			A.REI.1		
			A.REI.3		
			A.REI.10 (linear)		
			A.REI.12		
Linear Functions		8.F.1	F.IF.1		F.BF.4a
		8.F.2	F.IF.2 (linear)		F.LE.5
		8.F.3	F.IF.3 (linear)		F.LE.1.b
		8.F.4	F.IF.4b (linear)		F.LE.2
		8.F.5	F.IF.5b (linear)		
			F.IF.7a		
			F.IF.9b (linear)		
			F.BF.1ai (linear)		
			F.BF.2 (arithmetic)		
			F.LE.5 (linear)		
Angles	7.G.2	8.G.5			
	7.G.5				

Topic	Year 1			Year 2	
Description	(Grade 7, part of grade 8, equations and functions of Algebra 1)			(Part of grade 8 and the rest Algebra 1)	
Test	Grade 7 Test			Algebra 1 Test	
Area, Surface Area, and Volume	7.G.3		A.CED.4		
	7.G.4				
	7.G.6				
	8.G.9				
Transformations				8.G.1	
				8.G.2	
				8.G.3	
				8.G.4	
Univariate Statistics	7.SP.1		S.ID.1		
	7.SP.2		S.ID.2		
	7.SP.3		S.ID.3		
Probability	7.SP.5	8.SP.4	S.ID.5		
	7.SP.6				
	7.SP.7				
	7.SP.8				
The Real Number System including square roots, negative exponents, and scientific notation				8.NS.1	N.Q.1
				8.NS.2	N.Q,2
				8.EE.1	N.Q.3
				8.EE.2	
				8.EE.3	
				8.EE.4	
				8.G.6	
				8.G.7	
				8.G.8	
Bivariate Data				8.SP.1	S.ID.6
				8.SP.2	S.ID.7
				8.SP.3	S.ID.8

Topic	Year 1			Year 2	
Description	(Grade 7, part of grade 8, equations and functions of Algebra 1)			(Part of grade 8 and the rest Algebra 1)	
Test	Grade 7 Test			Algebra 1 Test	
Exponential and Quadratic Expressions, and Equations (Revisiting some linear functions and incorporating a few other function types when relevant.)					A.SSE.1
					A.SSE.2
					A.SSE.3abc
					A.APR.1a
					A.CED.1ab
					A.CED.2ab
					A.CED.3
					A.CED.4a
					A.REI.1
					A.REI.4ab
					A.REI.7
					A.REI.10
					A.REI.11
Exponential and Quadratic Functions (Revisiting some linear functions and incorporating a few other function types when relevant.)					F.IF.2
					F.IF.3
					F.IF.4b
					F.IF.5b
					F.IF.7be
					F.IF.8ab
					F.IF.9b
					F.BF.1a
					F.BF.2
					F.BF.3a
					F.BF.4a
					F.LE.1abc
					F.LE.2
					F.LE.3
					F.LE.5

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