Ohio's K-5 Learning Progressions

Counting and Cardinality

Kindergarten

Know number names and the count sequence.
1. Count to 100 by ones and by tens.
2. Count forward within 100 beginning from any given number other than 1.
3. Write numerals from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).
Count to tell the number of objects.
4. Understand the relationship between numbers and quantities; connect counting to cardinality using a variety of objects including pennies.
a. When counting objects, establish a one-to-one relationship by saying the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
c. Understand that each successive number name refers to a quantity that is one larger.
5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle; or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.
Compare numbers.
6. Orally identify (without using inequality symbols) whether the number of objects in one group is greater/more than, less/fewer than, or the same as the number of objects in another group, not to exceed 10 objects in each group.
7. Compare (without using inequality symbols) two numbers between 0 and 10 when presented as written numerals.

Ohio's K-5 Learning Progressions

Red Addition and Subtraction
Blue Multiplication and Division
Black Number
Brown Geometry

Number and Operations in Base Ten

Kindergarten	Grade One	Grade Two	Grade Three	Grade Four	Grade 5
Work with numbers 11-19 to gain foundations for place value. **1.** Compose and decompose numbers from 11 to 19 into a group of ten ones and some further ones by using objects and, when appropriate, drawings or equations; understand that these numbers are composed of a group of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.	**Extend the counting sequence.** **1.** Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral. **Understand place value.** **2.** Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: 10 can be thought of as a bundle of ten ones - called a "ten;" the numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones; and the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). **3.** Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. **Use place value understanding and properties of operations to add and subtract.** **4.** Add within 100, including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship	**Understand place value.** **1.** Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: **a.** 100 can be thought of as a bundle of ten tens — called a "hundred." **b.** The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones). **2.** Count forward and backward within 1,000 by ones, tens, and hundreds starting at any number; skip-count by 5s starting at any multiple of 5. **3.** Read and write numbers to 1,000 using base-ten numerals, number names, expanded form, and equivalent representations, e.g., 716 is 700 + 10 + 6, or 6 + 700 + 10, or 6 ones and 71 tens, etc. **4.** Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons. **Use place value understanding and properties of operations to add and subtract.** **5.** Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the	**Use place value understanding and properties of operations to perform multi-digit arithmetic. A range of strategies and algorithms may be used.** **1.** Use place value understanding to round whole numbers to the nearest 10 or 100. **2.** Fluently add and subtract within 1,000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. **3.** Multiply one-digit whole numbers by multiples of 10 in the range 10-90, e.g., 9 × 80, 5 × 60 using strategies based on place value and properties of operations.	**Generalize place value understanding for multi-digit whole numbers less than or equal to 1,000,000.** **1.** Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right by applying concepts of place value, multiplication, or division. **2.** Read and write multi-digit whole numbers using standard form, word form, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000. **3.** Use place value understanding to round multi-digit whole numbers to any place through 1,000,000. **Use place value understanding and properties of operations to perform multi-digit arithmetic with whole numbers less than or equal to 1,000,000.** **4.** Fluently add and subtract multi-digit whole numbers using a standard algorithm. **5.** Multiply a whole number of up to four digits by a one-digit whole	**Understand the place value system.** **1.** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $^1/_{10}$ of what it represents in the place to its left. **2.** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10. **3.** Read, write, and compare decimals to thousandths. **a.** Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × $(^1/_{10})$ + 9 × $(^1/_{100})$ + 2 × $(^1/_{1000})$. **b.** Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons. **4.** Use place value understanding to round decimals to any place, millions through hundredths. **Perform operations with multi-digit whole numbers and with decimals to hundredths.** **5.** Fluently multiply multi-digit whole numbers using a standard algorithm.

Ohio's K-5 Learning Progressions

Red Addition and Subtraction
Blue Multiplication and Division
Black Number
Brown Geometry

Number and Operations in Base Ten

Kindergarten	Grade One	Grade Two	Grade Three	Grade Four	Grade 5
	between addition and subtraction; record the strategy with a written numerical method (drawings and, when appropriate, equations) and explain the reasoning used. Understand that when adding two-digit numbers, tens are added to tens; ones are added to ones; and sometimes it is necessary to compose a ten. **5.** Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used. **6.** Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	relationship between addition and subtraction. **6.** Add up to four two-digit numbers using strategies based on place value and properties of operations. **7.** Add and subtract within 1,000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; record the strategy with a written numerical method (drawings and, when appropriate, equations) and explain the reasoning used. Understand that in adding or subtracting three-digit numbers, hundreds are added or subtracted from hundreds, tens are added or subtracted from tens, ones are added or subtracted from ones; and sometimes it is necessary to compose or decompose tens or hundreds. **8.** Mentally add 10 or 100 to a given number 100-900, and mentally subtract 10 or 100 from a given number 100-900. **9.** Explain why addition and subtraction strategies work, using place value and the properties of operations. Explanations may be supported by drawings or objects.		number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **6.** Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	**6.** Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. **7.** Solve real-world problems by adding, subtracting, multiplying, and dividing decimals using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction, or multiplication and division; relate the strategy to a written method and explain the reasoning used. **a.** Add and subtract decimals, including decimals with whole numbers, (whole numbers through the hundreds place and decimals through the hundredths place). **b.** Multiply whole numbers by decimals (whole numbers through the hundreds place and decimals through the hundredths place). **c.** Divide whole numbers by decimals and decimals by whole numbers (whole numbers through the tens place and decimals less than one through the hundredths place using numbers that work well with one another). *For example, 0.75 divided by 5, 18 divided by 0.6, or 0.9 divided by 3.*

Ohio's K-5 Learning Progressions

Red Addition and Subtraction
Blue Multiplication and Division
Black Number
Brown Geometry

Number and Operations - Fractions

Grade Three

Develop understanding of fractions as numbers. Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

1. Understand a fraction $^1/_b$ as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction $^a/_b$ as the quantity formed by *a* parts of size $^1/_b$.

2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction $^1/_b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size $^1/_b$ and that the endpoint of the part based at 0 locates the number $^1/_b$ on the number line.

b. Represent a fraction $^a/_b$ (which may be greater than 1) on a number line diagram by marking off *a* lengths $^1/_b$ from 0. Recognize that the resulting interval has size $^a/_b$ and that its endpoint locates the number $^a/_b$ on the number line.

3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size or on the same point on a number line.

b. Recognize and generate simple equivalent fractions, e.g., $^1/_2 = {}^2/_4$, $^4/_6 = {}^2/_3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = $^3/_1$; recognize that $^6/_1 = 6$; locate $^4/_4$ and 1 at the same point of a number line diagram.*

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size.

Grade Four

Extend understanding of fraction equivalence and ordering limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

1. Explain why a fraction $^a/_b$ is equivalent to a fraction $^{(n \times a)}/_{(n \times b)}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $^1/_2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. (Fractions need not be simplified.)

3. Understand a fraction $^a/_b$ with $a > 1$ as a sum of fractions $^1/_b$.

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* $^3/_8 = {}^1/_8 + {}^1/_8 + {}^1/_8$; $^3/_8 = {}^1/_8 + {}^2/_8$; $2\ {}^1/_8 = 1 + 1 + {}^1/_8 = {}^8/_8 + {}^8/_8 + {}^1/_8$.

c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction $^a/_b$ as a multiple of $^1/_b$. *For example, use a visual fraction model to represent $^5/_4$ as the product $5 \times (^1/_4)$, recording the conclusion by the equation $^5/_4 = 5 \times (^1/_4)$, or $^5/_4 = (^1/_4) + (^1/_4) + (^1/_4) + (^1/_4) + (^1/_4)$.*

b. Understand a multiple of $^a/_b$ as a multiple of $^1/_b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to*

Grade 5

Use equivalent fractions as a strategy to add and subtract fractions (Fractions need not be simplified.)

1. Add and subtract fractions with unlike denominators (including mixed numbers and fractions greater than 1) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, use visual models and properties of operations to show $^2/_3 + {}^5/_4 = {}^8/_{12} + {}^{15}/_{12} = {}^{23}/_{12}$. In general, $^a/_b + {}^c/_d = (^a/_b \times {}^d/_d) + (^c/_d \times {}^b/_b) = {}^{(ad+bc)}/_{bd}$.*

2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result $^2/_5 + {}^1/_2 = {}^3/_7$, by observing that $^3/_7 < {}^1/_2$.*

Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (Fractions need not be simplified.)

3. Interpret a fraction as division of the numerator by the denominator ($^a/_b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret $^3/_4$ as the result of dividing 3 by 4, noting that $^3/_4$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $^3/_4$. If 9 people want to share a 50 pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product $(^a/_b) \times q$ as a parts of a partition of *q* into *b* equal parts, equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show $(^2/_3) \times 4 = {}^8/_3$, and create a story context for this equation. Do the same with $(^2/_3) \times (^4/_5) = {}^8/_{15}$. (In general, $(^a/_b) \times (^c/_d) = {}^{ac}/_{bd}$.)*

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5. Interpret multiplication as scaling (resizing).

Ohio's K-5 Learning Progressions

Red Addition and Subtraction
Blue Multiplication and Division
Black Number
Brown Geometry

Number and Operations - Fractions

Grade Three	Grade Four	Grade 5
Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	*express $3 \times (^2/_5)$ as $6 \times (^1/_5)$, recognizing this product as $^6/_5$. (In general, $n \times (^a/_b) = (^{n \times a})/_b$.)* **c.** Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat $^3/_8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?* **Understand decimal notation for fractions, and compare decimal fractions limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.** **5.** Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express $^3/_{10}$ as $^{30}/_{100}$, and add $^3/_{10} + ^4/_{100} = ^{34}/_{100}$.* In general, students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators, but addition and subtraction with unlike denominators is not a requirement at this grade. **6.** Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as $^{62}/_{100}$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.* **7.** Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	**a.** Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. **b.** Explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $^a/_b = (^{n \times a})/_{(n \times b)}$ to the effect of multiplying $^a/_b$ by 1. **6.** Solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. **7.** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. In general students able to multiply fractions can develop strategies to divide fractions, by reasoning about the relationship between multiplication and division, but division of a fraction by a fraction is not a requirement at this grade. **a.** Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for $(^1/_3) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(^1/_3) \div 4 = (^1/_{12})$ because $(^1/_{12}) \times 4 = (^1/_3)$.* **b.** Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for $4 \div (^1/_5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (^1/_5) = 20$ because $20 \times (^1/_5) = 4$.* **c.** Solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share $^1/_2$ pound of chocolate equally? How many $^1/_3$ cup servings are in 2 cups of raisins?*

Ohio's K-5 Learning Progressions

Red Addition and Subtraction
Blue Multiplication and Division
Black Number
Brown Geometry

Operations and Algebraic Thinking

Kindergarten	Grade One	Grade Two	Grade Three	Grade Four	Grade Five
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from. **1.** Represent addition and subtraction with objects, fingers, mental images, drawings, sounds such as claps, acting out situations, verbal explanations, expressions, or equations. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.) **2.** Solve addition and subtraction problems (written or oral), and add and subtract within 10 by using objects or drawings to represent the problem. **3.** Decompose numbers and record compositions for numbers less than or equal to 10 into pairs in more than one way by using objects and, when appropriate, drawings or equations.	**Represent and solve problems involving addition and subtraction.** **1.** Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. See Glossary, Table 1. **2.** Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.) **Understand and apply properties of operations and the relationship between addition and subtraction.** **3.** Apply properties of operations as strategies to add and subtract. *For example, if 8 + 3 = 11 is known, then 3 + 8 = 11 is also known (Commutative Property of Addition); to add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12 (Associative Property of Addition).* Students need not use formal terms for these properties. **4.** Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*	**Represent and solve problems involving addition and subtraction.** **1.** Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. See Glossary, Table 1. **Add and subtract within 20.** **2.** Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers. See standard 1.OA.6 for a list of mental strategies. **Work with equal groups of objects to gain foundations for multiplication.** **3.** Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.	**Represent and solve problems involving multiplication and division.** **1.** Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. (Note: These standards are written with the convention that *a* x *b* means *a* groups of *b* objects each; however, because of the commutative property, students may also interpret 5 x 7 as the total number of objects in 7 groups of 5 objects each). **2.** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* **3.** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. See Glossary, Table 2. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.) **4.** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example,*	**Use the four operations with whole numbers to solve problems.** **1.** Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. **2.** Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. See Glossary, Table 2. Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.) **3.** Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a	**Write and interpret numerical expressions.** **1.** Use parentheses in numerical expressions, and evaluate expressions with this symbol. Formal use of algebraic order of operations is not necessary. **2.** Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18,932 + 921) is three times as large as 18,932 + 921, without having to calculate the indicated sum or product.* **Analyze patterns and relationships.** **3.** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Ohio's K-5 Learning Progressions

Red Addition and Subtraction
Blue Multiplication and Division
Black Number
Brown Geometry

Operations and Algebraic Thinking

Kindergarten	Grade One	Grade Two	Grade Three	Grade Four	Grade Five
4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or, when appropriate, an equation. **5.** Fluently add and subtract within 5.	**Add and subtract within 20.** **5.** Relate counting to addition and subtraction, e.g., by counting on 2 to add 2. **6.** Add and subtract within 20, demonstrating fluency with various strategies for addition and subtraction within 10. Strategies may include counting on; making ten, e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14; decomposing a number leading to a ten, e.g., 13 − 4 = 13 − 3 − 1 = 10 − 1 = 9; using the relationship between addition and subtraction, e.g., knowing that 8 + 4 = 12, one knows 12 − 8 = 4; and creating equivalent but easier or known sums, e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13. **Work with addition and subtraction equations.** **7.** Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false?* *6 = 6; 7 = 8 – 1; 5 + 2 = 2 + 5;* *4 + 1 = 5 + 2.* **8.** Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations: 8 + ☐ = 11; 5 = ☐ − 3;* *6 + 6 = ☐.*	**4.** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	*determine the unknown number that makes the equation true in each of the equations 8 × ☐ = 48, 5 = ☐ ÷ 3, 6 × 6 = ☐.* **Understand properties of multiplication and the relationship between multiplication and division.** **5.** Apply properties of operations as strategies to multiply and divide. *For example, if 6 × 4 = 24 is known, then 4 × 6 = 24 is also known (Commutative Property of Multiplication); 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30 (Associative Property of Multiplication); knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56 (Distributive Property).* Students need not use formal terms for these properties. **6.** Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.* **Multiply and divide within 100.** **7.** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division, e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8 or properties of operations. Limit to division without remainders. By the end of Grade 3, know from memory all products of two one-digit numbers. **Solve problems involving the four operations, and identify and explain patterns in arithmetic.**	letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. **Gain familiarity with factors and multiples.** **4.** Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite. **Generate and analyze patterns.** **5.** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*	

Ohio's K-5 Learning Progressions

Red Addition and Subtraction
Blue Multiplication and Division
Black Number
Brown Geometry

Operations and Algebraic Thinking

Kindergarten	Grade One	Grade Two	Grade Three	Grade Four	Grade Five
			8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter or a symbol, which stands for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. This standard is limited to problems posed with whole numbers and having whole-number answers. Students may use parentheses for clarification since algebraic order of operations is not expected. **9.** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*		

Ohio's K-5 Learning Progressions

Red Addition and Subtraction
Blue Multiplication and Division
Black Number
Brown Geometry

Geometry (K-5)

Kindergarten	Grade One	Grade Two	Grade Three	Grade Four	Grade 5
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres). **1.** Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*. **2.** Correctly name shapes regardless of their orientations or overall size. **3.** Identify shapes as two-dimensional (lying in a plane, "flat") or three dimensional ("solid"). **Describe, compare, create, and compose shapes.** **4.** Describe and compare two- or three-dimensional shapes, in different sizes and orientations, using informal language to describe their commonalities, differences, parts, and other attributes. **5.** Model shapes in the world by building shapes from components (such as sticks and clay balls) and drawing shapes. **6.** Combine simple shapes to form larger shapes.	**Reason with shapes and their attributes.** **1.** Distinguish between defining attributes, e.g., triangles are closed and three-sided, versus non-defining attributes, e.g., color, orientation, overall size; build and draw shapes that possess defining attributes. **2.** Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape. Students do not need to learn formal names such as "right rectangular prism." **3.** Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves, fourths,* and *quarters,* and use the phrases *half of, fourth of,* and *quarter of.* Describe the whole as two of or four of the shares in real-world contexts. Understand for these examples that decomposing into more equal shares creates smaller shares.	**Reason with shapes and their attributes.** **1.** Recognize and identify triangles, quadrilaterals, pentagons, and hexagons based on the number of sides or vertices. Recognize and identify cubes, rectangular prisms, cones, and cylinders. **2.** Partition a rectangle into rows and columns of same-size squares and count to find the total number of them. **3.** Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words *halves, thirds,* or *fourths* and *quarters,* and use the phrases *half of, third of,* or *fourth of* and *quarter of.* Describe the whole as two halves, three thirds, or four fourths in real-world contexts. Recognize that equal shares of identical wholes need not have the same shape.	**Reason with shapes and their attributes.** **1.** Draw and describe triangles, quadrilaterals (rhombuses, rectangles, and squares), and polygons (up to 8 sides) based on the number of sides and the presence or absence of square corners (right angles). **2.** Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as $^1/_4$ of the area of the shape.*	**Draw and identify lines and angles, and classify shapes by properties of their lines and angles.** **1.** Draw points, lines, line segments, rays, angles (right, acute, and obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. **2.** Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size.	**Graph points on the coordinate plane to solve real-world and mathematical problems.** **1.** Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond, e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate. **2.** Represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation. **Classify two-dimensional figures into categories based on their properties.** **3.** Identify and describe commonalities and differences of triangles based on angle measures (equiangular, right, acute, and obtuse triangles) and side lengths (isosceles, equilateral, and scalene triangles). **4.** Identify and describe commonalities and differences of quadrilaterals based on angle measures, side lengths, and the presence or absence of parallel and perpendicular lines, e.g., squares, rectangles, parallelograms, trapezoids, and rhombuses.

Ohio's K-5 Learning Progressions

Red Addition and Subtraction
Blue Multiplication and Division
Black Number
Brown Geometry

Measurement and Data

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Identify, describe, and compare measurable attributes. **1.** Identify and describe measurable attributes (length, weight, and height) of a single object using vocabulary terms such as long/short, heavy/light, or tall/short. **2.** Directly compare two objects with a measurable attribute in common to see which object has "more of" or "less of" the attribute, and describe the difference. *For example, directly compare the heights of two children, and describe one child as taller/shorter.* **Classify objects and count the number of objects in each category.** **3.** Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. The number of objects in each category should be less than or equal to ten. Counting and sorting coins	**Measure lengths indirectly and by iterating length units.** **1.** Order three objects by length; compare the lengths of two objects indirectly by using a third object. **2.** Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.* **Work with time and money.** **3.** Work with time and money. **a.** Tell and write time in hours and half-hours using analog and digital clocks. **b.** Identify pennies and dimes by name and value. **Represent and interpret data.**	**Measure and estimate lengths in standard units.** **1.** Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes. **2.** Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen. **3.** Estimate lengths using units of inches, feet, centimeters, and meters. **4.** Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. **Relate addition and subtraction to length.** **5.** Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same whole number units, e.g., by using drawings and equations	**Solve problems involving money and measurement and estimation of intervals of time, liquid volumes, and masses of objects.** **1.** Work with time and money. **a.** Tell and write time to the nearest minute. Measure time intervals in minutes (within 90 minutes). Solve real-world problems involving addition and subtraction of time intervals (elapsed time) in minutes, e.g., by representing the problem on a number line diagram or clock. **b.** Solve word problems by adding and subtracting within 1,000, dollars with dollars and cents with cents (not using dollars and cents simultaneously) using the $ and ¢ symbol appropriately (not including decimal notation). **2.** Measure and estimate liquid volumes and masses of objects using standard units of grams, kilograms, and liters. Add, subtract, multiply, or divide whole numbers to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. Excludes multiplicative comparison problems involving notions of "times as much"; see Glossary, Table 2. **Represent and interpret data.** **3.** Create scaled picture graphs to represent a data set with several categories. Create scaled bar graphs to represent a data set with several categories. Solve two-step "how many more" and "how many less" problems using information presented in the scaled graphs. *For example, create a bar graph in which each square in the bar graph might represent 5 pets,*	**Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.** **1.** Know relative sizes of the metric measurement units within one system of units. Metric units include kilometer, meter, centimeter, and millimeter; kilogram and gram; and liter and milliliter. Express a larger measurement unit in terms of a smaller unit. Record measurement conversions in a two-column table. *For example, express the length of a 4-meter rope in centimeters. Because 1 meter is 100 times as long as a 1 centimeter, a two-column table of meters and centimeters includes the number pairs 1 and 100, 2 and 200, 3 and 300,...* **2.** Solve real-world problems involving money, time, and metric measurement. **a.** Using models, add and subtract money and express the answer in decimal notation. **b.** Using number line diagrams, clocks, or other models, add and subtract intervals of time in hours and minutes. **c.** Add, subtract, and multiply whole numbers to solve metric measurement problems involving distances, liquid volumes, and masses of objects. **3.** Develop efficient strategies to determine the area and perimeter of rectangles in real-world situations and mathematical problems. *For example, given the total area and one side length of a rectangle, solve for the*	**Convert like measurement units within a given measurement system.** **1.** Know relative sizes of these U.S. customary measurement units: pounds, ounces, miles, yards, feet, inches, gallons, quarts, pints, cups, fluid ounces, hours, minutes, and seconds. Convert between pounds and ounces; miles and feet; yards, feet, and inches; gallons, quarts, pints, cups, and fluid ounces; hours, minutes, and seconds in solving multi-step, real-world problems. **Represent and interpret data.** **2.** Display and interpret data in graphs (picture graphs, bar graphs, and line plots) to solve problems using numbers and operations for this grade, e.g., including U.S. customary units in fractions $^1/_2$, $^1/_4$, $^1/_8$, or decimals. **Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.** **3.** Recognize volume as an attribute of solid figures and understand concepts of volume measurement. **a.** A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

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should be limited to pennies.	**4.** Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.	with a symbol for the unknown number to represent the problem. Drawings need not show details but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.) **6.** Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram. **Work with time and money.** **7.** Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. **8.** Solve problems with money. **a.** Identify nickels and quarters by name and value. **b.** Find the value of a collection of quarters, dimes, nickels, and pennies.	*then determine how many more/less in two given categories.* **4.** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by creating a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters. **Geometric measurement: understand concepts of area and relate area to multiplication and to addition.** **5.** Recognize area as an attribute of plane figures and understand concepts of area measurement. **a.** A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. **b.** A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units. **6.** Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). **7.** Relate area to the operations of multiplication and addition. **a.** Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. **b.** Multiply side lengths to find areas of rectangles with whole- number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.	*unknown factor, and given two adjacent side lengths of a rectangle, find the perimeter.* **Represent and interpret data.** **4.** Display and interpret data in graphs (picture graphs, bar graphs, and line plots) to solve problems using numbers and operations for this grade. **Geometric measurement: understand concepts of angle and measure angles.** **5.** Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint and understand concepts of angle measurement. **a.** Understand an angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $^1/_{360}$ of a circle is called a "one-degree angle," and can be used to measure angles. **b.** Understand angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees. **6.** Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure. **7.** Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical	**b.** A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units. **4.** Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units. **5.** Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume. **a.** Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the Associative Property of Multiplication. **b.** Apply the formulas $V = \ell \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real-world and mathematical problems. **c.** Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping

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		c. Solve word problems by adding and subtracting within 100, dollars with dollars and cents with cents (not using dollars and cents simultaneously) using the $ and ¢ symbols appropriately (not including decimal notation). **Represent and interpret data.** **9.** Generate measurement data by measuring lengths of several objects to the nearest whole unit or by making repeated measurements of the same object. Show the measurements by creating a line plot, where the horizontal scale is marked off in whole-number units. **10.** Organize, represent, and interpret data with up to four categories; complete picture graphs when single-unit scales are provided; complete bar graphs when single-unit scales are provided; solve simple put-together, take-apart, and compare problems in a graph. See Glossary, Table 1.	**c.** Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b* + *c* is the sum of *a* × *b* and *a* × *c* (represent the distributive property with visual models including an area model). **d.** Recognize area as additive. Find the area of figures composed of rectangles by decomposing into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems. **Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.** **8.** Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	problems, e.g., by using an equation with a symbol for the unknown angle measure.	parts, applying this technique to solve real- world problems.