Algebra 2 and Mathematics 3 Critical Areas of Focus - DRAFT

Ohio's Learning Standards for Mathematics include descriptions of the Conceptual Categories. These descriptions have been used to develop critical areas for each of the courses in both the Traditional and Integrated pathways. The critical areas are designed to bring focus to the standards in each course by describing the big ideas that educators can use to build their high school curriculum and to guide instruction. Each course contains up to six critical areas. This document identifies the clusters and standards that build toward each critical area.

The purpose of this document is to facilitate discussion among teachers and curriculum experts and to encourage coherence in the sequence, pacing and units of study for high school curriculum. Professional learning communities can use the following questions as examples to develop their high school curriculum.

DISCUSSION QUESTIONS

Example 1: Analyze and discuss the content for each high school course's Critical Areas of Focus.

What are the concepts?

What are the procedures and skills?

What are the key mathematical practices?

What are the relationships students are to make?

What further information is needed? For example, what does prove mean?

What are appropriate models for representing this learning?

Example 2: Identify and discuss the connections among the conceptual categories, domains, clusters and standards *within* each course's Critical Areas of Focus.

What are the relationships among the conceptual categories, domains, clusters and standards?

Why is each relationship important?

What are the differences?

How does the Critical Area of Focus description inform the instruction of the related conceptual categories, domains, clusters and standards?

Example 3: Identify and discuss any connections *across* the Critical Areas of Focus within a course. This information will help create a sequence of units for the course.

Example 4: Compare each Critical Area of Focus to those for the preceding and succeeding courses to become familiar with previous and future learning.

What understandings does this learning build upon?

What are the related future understandings?

Example 5: Compare and contrast Ohio's Learning Standards to the current district curriculum.

What is taught now but not in Ohio's Learning Standards?

What content is essentially the same? Identify the differences.

What will be new content for this grade?

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CRITICAL AREA OF FOCUS #1

Communication and Analysis

Within this critical area students develop conclusions based on quantitative information and critical thinking. They recognize, make, and evaluate underlying assumptions in estimation, modeling, and data analysis. Students then organize and present thoughts and processes using mathematical and statistical evidence. They communicate clear and complete information in such a way that the reader or listener can understand the contextual and quantitative information in a situation. Students demonstrate numerical reasoning orally and in writing coherent statements and paragraphs.

In the context of real-world applications, students make and investigate mathematical conjectures. They are able to defend their conjectures and respectfully question conjectures made by their classmates. This leads to the development of mathematical arguments and informal proofs, which are ways of expressing particular kinds of reasoning and justification. Explanations (oral and written) include mathematical arguments and rationales, not just procedural descriptions or summaries. Listening to others' explanations gives students opportunities to develop their own understandings. Through communication, ideas become objects of reflection, refinement, discussion, and amendment. When students are challenged to communicate the results of their thinking to others orally or in writing, they learn to be clear, convincing, and precise in their use of mathematical language. Additionally, conversations in which mathematical ideas are explored from multiple perspectives help the participants sharpen their thinking and make connections. **This critical area of focus cross cuts all the other critical areas of focus.**

This critical area cross cuts all the rest of the standards, so all standards in this document also fall under this crucial area!!!

Algebra 2 and Mathematics 3 Critical Areas of Focus

CRITICAL AREA OF FOCUS #2

Modeling with Functions

The heart of this course is illustrated in the description of modeling as “the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions.” The narrative discussion and diagram of the modeling cycle should be applied in context.

Students extend their work with linear, exponential, and quadratic functions to include additional function families, such as higher degree polynomial, rational, radical, logarithmic, and absolute value functions, as well as a more in-depth look at step and piecewise defined functions.

Students synthesize and generalize what they have learned about a variety of function families. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying functions. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit.

Functions – Interpreting Functions

Interpret functions that arise in applications in terms of the context.

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ (A2, M3)

Interpret functions that arise in applications in terms of the context.

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

- **c.** Emphasize the selection of a type of function for a model based on behavior of data and context. (A2, M3)

F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★ (A2, M3)

Algebra 2 and Mathematics 3 Critical Areas of Focus

CRITICAL AREA OF FOCUS #2, CONTINUED

Modeling with Functions

Functions – Interpreting Functions

Analyze functions using different representations.

F.IF.7 Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.★

- **c.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. (A2, M3)
- **d.** Graph polynomial functions, identifying zeros, when factoring is reasonable, and indicating end behavior. (A2, M3)
- **f.** Graph exponential functions, indicating intercepts and end behavior, and trigonometric functions, showing period, midline[G], and amplitude. (A2, M3)
- **g.** Graph rational functions, identifying zeros and asymptotes when factoring is reasonable, and indicating end behavior. (A2, M3)
- **h.** Graph logarithmic functions, indicating intercepts and end behavior. (A2, M3)

F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- **a.** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (A2, M3)
- **b.** Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as $y = (1.02)^t$, and $y = (0.97)^t$ and classify them as representing exponential growth or decay.* (A2, M3)

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (A2, M3)

Functions – Building Functions

Build a function that models a relationship between two quantities.

F.BF.1 Write a function that describes a relationship between two quantities.★

- **b.** Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential and relate these functions to the model.* (A2, M3)
- **c.** Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

Algebra 2 and Mathematics 3 Critical Areas of Focus

CRITICAL AREA OF FOCUS #2, CONTINUED

Modeling with Functions

Functions – Building Functions

Build new functions from existing functions.

F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (A2, M3)

F.BF.4 Find inverse functions.

- **b.** Read values of an inverse function from a graph or a table, given that the function has an inverse. (A2, M3)
- **c**. Verify by composition that one function is the inverse of another. (A2, M3)
- **d.** Find the inverse of a function algebraically, given that the function has an inverse. (A2, M3)

Functions – Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models, and solve problems.

F.LE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.★

Statistics and Probability – Interpreting Categorical and Quantitative Data

Summarize, represent, and interpret data on two categorical and quantitative variables.

S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. ★

- **a.** Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions, or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* (A2, M3)
- **b.** Informally assess the fit of a function by discussing residuals. (A2, M3)

Algebra 2 and Mathematics 3 Critical Areas of Focus

CRITICAL AREA OF FOCUS #3

Extending Algebraic Reasoning

In this course, students extend their understanding of previous topics to include more advanced structures, e.g., from linear and quadratic to general polynomials; from exponentials with integer domains to those with rational domains; from the real number system to the complex number system.

For example, students build on their previous work with rational numbers and properties of integer exponents to develop understanding of rational exponents. They extend this new understanding of rational exponents to explore irrational numbers and radical relationships.

Number and Quantity – The Real Number System

Extend the properties of exponents to rational exponents.

N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*

N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers.

N.RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Number and Quantity – The Complex Number System

Perform arithmetic operations with complex numbers.

N.CN.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

N.CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

N.CN.3 Find the conjugate of a complex number; use conjugates to find magnitudes and quotients of complex numbers.

Algebra – Arithmetic with Polynomials and Rational Expressions

Perform arithmetic operations on polynomials.

A.APR.1 Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

- **b.** Extend to polynomial expressions beyond those expressions that simplify to forms that are linear or quadratic. (A2, M3)

Algebra 2 and Mathematics 3 Critical Areas of Focus

CRITICAL AREA OF FOCUS #3, CONTINUED

Extending Algebraic Reasoning

Algebra – Arithmetic with Polynomials and Rational Expressions

Rewrite rational expressions.

A.APR.7 Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Algebra 2 and Mathematics 3 Critical Areas of Focus

CRITICAL AREA OF FOCUS #4

Polynomial and Rational Relationships

Students develop the structural similarities between the system of polynomials and the system of integers. They draw on analogies between polynomial arithmetic and base ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. This learning culminates with applying the Fundamental Theorem of Algebra.

A central theme of the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers. Students realize that rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, they learn that rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial.

Number and Quantity – The Complex Number System

Use complex numbers in polynomial identities and equations.

N.CN.7 Solve quadratic equations with real coefficients that have complex solutions.

N.CN.8 Extend polynomial identities to the complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.*

N.CN.9 Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Algebra – Seeing Structure in Expressions

Interpret the structure of expressions.

A.SSE.1. Interpret expressions that represent a quantity in terms of its context. ★

- **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
- **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity.

A.SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, to factor $3x(x - 5) + 2(x - 5)$, students should recognize that the "$x - 5$" is common to both expressions being added, so it simplifies to $(3x + 2)(x - 5)$; or see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored a $(x^2 - y^2)(x^2 + y^2)$.*

Write expressions in equivalent forms to solve problems.

A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★

- **c.** Use the properties of exponents to transform expressions for exponential functions. *For example, 8^t can be written as 2^{3t}.*

(+) **A.SSE.4** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1) and use the formula to solve problems. *For example, calculate mortgage payments.* ★

Algebra 2 and Mathematics 3 Critical Areas of Focus

CRITICAL AREA OF FOCUS #4, CONTINUED

Polynomial and Rational Relationships

Algebra – Arithmetic with Polynomials and Rational Expressions

Understand the relationship between zeros and factors of polynomials.

A.APR.2 Understand and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$. In particular, $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

A.APR.3 Identify zeros of polynomials, when factoring is reasonable, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems.

(+) **A.APR.5** Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers. *For example, by using coefficients determined for by Pascal's Triangle.* The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

Rewrite rational expressions.

A.APR.6 Rewrite simple rational expressions in different forms; write ${}^{a(x)}/_{b(x)}$ in the form $q(x) + {}^{r(x)}/_{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

Rewrite rational expressions.

A.APR.7 Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Algebra – Creating Equations

Create equations that describe numbers or relationships.

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions.*★

- **c.** Extend to include more complicated function situations with the option to solve with technology. (A2, M3)

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★

- **c.** Extend to include more complicated function situations with the option to graph with technology. (A2, M3)

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* ★ (A1, M1)

- **a.** While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)

Algebra 2 and Mathematics 3 Critical Areas of Focus

CRITICAL AREA OF FOCUS #4, CONTINUED

Polynomial and Rational Relationships

Algebra – Creating Equations

Create equations that describe numbers or relationships.

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.★

d. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)

Algebra - Reasoning with Equations and Inequalities

Understand solving equations as a process of reasoning and explain the reasoning.

A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve systems of equations.

A.REI.6 Solve systems of linear equations algebraically and graphically.

(+) b. Extend to include solving systems of linear equations in three variables, but only algebraically (A2, M3)

Represent and solve equations and inequalities graphically.

A.REI.11 Explain why the x-coordinates of the points where the graphs of the equation $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.

Algebra 2 and Mathematics 3 Critical Areas of Focus

Critical Area of Focus #5

Trigonometry of General Triangles

In Geometry or Mathematics 2, students used trigonometric ratios to find lengths of sides in right triangles. In Algebra 2 or Mathematics 3, students extend that knowledge including the use of inverse operations to find angles and solve problems involving trigonometry and the Pythagorean theorem. To enrich the study of triangle trigonometry, students explore the Laws of Sines and Cosines to find missing measures of general (not necessarily right) triangles.

Geometry – Similarity, Right Triangles, and Trigonometry

Define trigonometric ratios, and solve problems involving right triangles.

G.SRT.8 Solve problems involving right triangles.★

- **b.** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. ★ (A2, M3)

Note: Standard G.SRT Part a. is included in this course to be considered for next standards revision

Apply trigonometry to general triangles.

G.SRT.9 Derive the formula $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

G.SRT.10 Explain proofs of the Laws of Sines and Cosines and use the Laws to solve problems.

- ***a.*** *Extend right triangle trigonometry to include obtuse angles.*

G.SRT.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles, e.g., surveying problems, resultant forces.

Functions – Building Functions

Build new functions from existing functions.

F.BF.4 Find inverse functions.

- **b.** Read values of an inverse function from a graph or a table, given that the function has an inverse. (A2, M3)
- **c.** Verify by composition that one function is the inverse of another. (A2, M3)
- **d.** Find the inverse of a function algebraically, given that the function has an inverse. (A2, M3)