Algebra 1 Critical Areas of Focus

Ohio's Learning Standards for Mathematics include descriptions of the Conceptual Categories. These descriptions have been used to develop critical areas for each of the courses in both the Traditional and Integrated pathways. The critical areas are designed to bring focus to the standards in each course by describing the big ideas that educators can use to build their high school curriculum and to guide instruction. Each course contains up to six critical areas. This document identifies the clusters and standards that build toward each critical area.

The purpose of this document is to facilitate discussion among teachers and curriculum experts and to encourage coherence in the sequence, pacing and units of study for high school curriculum. Professional learning communities can use the following questions as examples to develop their high school curriculum.

DISCUSSION QUESTIONS

Example 1: Analyze and discuss the content for each high school course's Critical Areas of Focus.
- What are the concepts?
- What are the procedures and skills?
- What are the key mathematical practices?
- What are the relationships students are to make?
- What further information is needed? For example, what does prove mean?
- What are appropriate models for representing this learning?

Example 2: Identify and discuss the connections among the conceptual categories, domains, clusters and standards <u>within</u> each course's Critical Areas of Focus.
- What are the relationships among the conceptual categories, domains, clusters and standards?
- Why is each relationship important?
- What are the differences?
- How does the Critical Area of Focus description inform the instruction of the related conceptual categories, domains, clusters and standards?

Example 3: Identify and discuss any connections <u>across</u> the Critical Areas of Focus within a course. This information will help create a sequence of units for the course. For example, the learning in Critical Area of Focus #2 should occur prior to Critical Area of Focus #5.

Example 4: Compare each Critical Area of Focus to those for the preceding and succeeding courses to become familiar with previous and future learning.
- What understandings does this learning build upon?
- What are the related future understandings?

Example 5: Compare and contrast Ohio's Learning Standards to the current district curriculum.
- What is taught now but not in Ohio's Learning Standards?
- What content is essentially the same? Identify the differences.
- What will be new content for this grade?

Algebra 1 Table of Contents

Algebra 1 Critical Areas of Focus

CRITICAL AREA OF FOCUS #1

Relationships Between Quantities and Reasoning with Equations

By the end of eighth grade students have learned to solve linear equations in one variable and have applied graphical methods to analyze and solve systems of linear equations in two variables. Now students build on these earlier experiences by analyzing and explaining the process of solving an equation. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. All this work is grounded on understanding quantities and on the relationships between them. Students apply this learning in real-world and modeling situations.

Number and Quantity - Quantities

Reason quantitatively and use units to solve problems.

N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.★

N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★

Algebra - Seeing Structure in Expressions

Interpret the structure of expressions.

A.SSE.1. Interpret expressions that represent a quantity in terms of its context. ★

- **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
- **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity.

Algebra - Creating Equations

Create equations that describe numbers or relationships.

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions. ★

- **a.** Focus on applying linear and simple exponential expressions. (A1, M1)
- **b.** Focus on applying simple quadratic expressions. (A1, M2)

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★

- **a.** Focus on applying linear and simple exponential expressions. (A1, M1)
- **b.** Focus on applying simple quadratic expressions. (A1, M2)

A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.* ★ (A1, M1)

CRITICAL AREA OF FOCUS #1, CONTINUED

Relationships Between Quantities and Reasoning with Equations

Algebra - Creating Equations

Create equations that describe numbers or relationships.

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. ★

a. Focus on formulas in which the variable of interest is linear or square. For example, rearrange Ohm's law V = IR to highlight resistance R, or rearrange the formula for the area of a circle $A = (\pi)r^2$ to highlight radius r. (A1)

Understand solving equations as a process of reasoning and explain the reasoning.

A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable.

A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Algebra 1 Critical Areas of Focus

CRITICAL AREA OF FOCUS #2

Linear and Exponential Relationships

In earlier grades, students define, evaluate, and compare functions, and use them to model relationships between quantities. Students will learn function notation and develop the concepts of domain and range. Their understanding moves beyond viewing functions as processes that take inputs and yield outputs and to viewing functions as objects in their own right followed by an informal introduction of inverse functions. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate or incomplete. Their work includes functions that can be described or modeled by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Algebra – Reasoning with Equations and Inequalities

Solve systems of equations.

A.REI.5 Verify that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A.REI.6 Solve systems of linear equations algebraically and graphically.

a. Limit to pairs of linear equations in two variables. (A1, M1)

Represent and solve equations and inequalities graphically.

A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A.REI.11 Explain why the x-coordinates of the points where the graphs of the equation $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.

A.REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

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CRITICAL AREA OF FOCUS #2, CONTINUED

Linear and Exponential Relationships

Functions – Interpreting Functions

Understand the concept of a function, and use function notation.

F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*.

F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n + 1) = f(n) + f(n − 1) for n ≥ 1.*

Functions – Interpreting Functions

Interpret functions that arise in applications in terms of the context.

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★ (A2, M3)

b. Focus on linear, quadratic, and exponential functions. (A1, M2)

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. ★

b. Focus on linear, quadratic, and exponential functions. (A1, M2)

Analyze functions using different representations.

F.IF.7 Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. ★

a. Graph linear functions and indicate intercepts. (A1, M1)

b. Graph quadratic functions and indicate intercepts, maxima, and minima. (A1, M2)

e. Graph simple exponential functions, indicating intercepts and end behavior. (A1, M1)

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (A2, M3)

b. Focus on linear, quadratic, and exponential functions. (A1, M2)

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CRITICAL AREA OF FOCUS #2, CONTINUED

Linear and Exponential Relationships

Functions – Building Functions

Build a function that models a relationship between two quantities.

F.BF.1 Write a function that describes a relationship between two quantities. ★
- **a.** Determine an explicit expression, a recursive process, or steps for calculation from a context.
  - **i.** Focus on linear and exponential functions. (A1, M1)
  - **ii.** Focus on situations that exhibit quadratic or exponential relationships. (A1, M2)

F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ★

Build new functions from existing functions.

F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (A2, M3)
- **a.** Focus on transformations of graphs of quadratic functions, except for $f(kx)$; (A1, M2)

F.BF.4 Find inverse functions.
- **a.** Informally determine the input of a function when the output is known. (A1, M1)

Functions – Linear, Quadratic, and Exponential Models

Construct and compare linear, quadratic, and exponential models, and solve problems.

F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. ★
- **a.** Show that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
- **b.** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- **c.** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). ★

F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. ★ (A1, M2)

Interpret expressions for functions in terms of the situation they model.

F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context. ★

Algebra 1 Critical Areas of Focus

CRITICAL AREA OF FOCUS #3

Descriptive Statistics

In middle school, students developed an understanding of statistical problem solving through the format of the GAISE Model. They were expected to display numerical data and summarize it using measures of center and variability. By the end of middle school, students were creating scatterplots and recognizing linear trends in data. Now, they apply those concepts by using the GAISE model in the context of real-world applications. Students develop formal means of assessing how a model fits data. They use regression techniques to describe approximately linear relationships between quantities. Students use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. In Algebra 2/Mathematics 3, students will look at residuals to analyze the goodness of fit.

Statistics and Probability

Summarize, represent, and interpret data on a single count or measurement variable

S.ID.1 Represent data with plots on the real number line (dot plots[G], histograms, and box plots) in the context of real-world applications using the GAISE model. ★

S.ID.2 In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviation, interquartile range, and standard deviation) of two or more different data sets. ★

S.ID.3 In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★

Summarize, represent, and interpret data on two categorical and quantitative variables.

S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in The data. ★

S.ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. ★

c. Fit a linear function for a scatterplot that suggests a linear association. (A1, M1)

Interpret linear models.

S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. ★

S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit. ★

Algebra 1 Critical Areas of Focus

CRITICAL AREA OF FOCUS #4

Expressions and Equations

Students apply the properties of operations with real numbers, the relationships between the operations, along with the properties of exponents to operations with polynomials. Also, students focus on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent. They create and solve equations, inequalities, and systems of equations involving exponential and quadratic expressions.

Algebra – Seeing the Structure in Expressions

Interpret the structure of expressions.

A.SSE.1. Interpret expressions that represent a quantity in terms of its context. ★

- **a.** Interpret parts of an expression, such as terms, factors, and coefficients.
- **b.** Interpret complicated expressions by viewing one or more of their parts as a single entity.

A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, to factor $3x(x - 5) + 2(x - 5)$, students should recognize that the "$x - 5$" is common to both expressions being added, so it simplifies to $(3x + 2)(x - 5)$; or see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems.

A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★

- **a.** Factor a quadratic expression to reveal the zeros of the function it defines.
- **b.** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- **c.** Use the properties of exponents to transform expressions for exponential functions. *For example, 8^t can be written as 2^{3t}.*

Algebra – Arithmetic with Polynomials and Rational Expressions

Perform arithmetic operations on polynomials.

A.APR.1 Understand that polynomials form a system analogous to the integers, namely, that they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

- **a.** Focus on polynomial expressions that simplify to forms that are linear or quadratic. (A1, M2)

Algebra – Creating Equations

Create equations that describe numbers or relationships.

A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions. ★

- **a.** Focus on applying linear and simple exponential expressions. (A1, M1)
- **b.** Focus on applying simple quadratic expressions. (A1, M2)

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★

- **a.** Focus on applying linear and simple exponential expressions. (A1, M1)
- **b.** Focus on applying simple quadratic expressions. (A1, M2)

CRITICAL AREA OF FOCUS #4, CONTINUED

Expressions and Equations

Algebra – Creating Equations

Create equations that describe numbers or relationships.

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. ★

- **a.** Focus on formulas in which the variable of interest is linear or square. For example, rearrange Ohm's law V = IR to highlight resistance R, or rearrange the formula for the area of a circle $A = (\pi)r^2$ to highlight radius r. (A1)

Algebra – Reasoning with Equations and Inequalities

Solve equations and inequalities in one variable.

A.REI.4 Solve quadratic equations in one variable.

- **a.** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions.
- **b.** Solve quadratic equations as appropriate to the initial form of the equation by inspection, e.g., for $x^2 = 49$; taking square roots; completing the square; applying the quadratic formula; or utilizing the Zero-Product Property after factoring.

(+) **c.** Derive the quadratic formula using the method of completing the square.

Solve systems of equations.

A.REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.*

CRITICAL AREA OF FOCUS #5

Quadratic Functions and Modeling

In preparation for work with quadratic relationships students explore distinctions between rational and irrational numbers. They consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to gather information about quadratic and exponential functions by interpreting various forms of expressions representing the functions. For example, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. When quadratic equations do not have real solutions, students learn that the graph of the related quadratic function does not cross the horizontal axis. They relate their prior experience with transformations to that of building new functions from existing ones and recognize the effect of the transformations on the graphs. Formal work with complex numbers and more specialized functions—absolute value, step, and piecewise-defined, will occur in Algebra 2/Mathematics 3.

Functions – Interpreting Functions

Interpret functions that arise in applications in terms of the context.

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★ (A2, M3)

b. Focus on linear, quadratic, and exponential functions. (A1, M2)

F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. ★

b. Focus on linear, quadratic, and exponential functions. (A1, M2)

F.IF.7 Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. ★

a. Graph linear functions and indicate intercepts. (A1, M1)

b. Graph quadratic functions and indicate intercepts, maxima, and minima. (A1, M2)

e. Graph simple exponential functions, indicating intercepts and end behavior. (A1, M1)

F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. (A2, M3)

i. Focus on completing the square to quadratic functions with the leading coefficient of 1. (A1)

b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change[G] in functions such as $y = (1.02)^t$, and $y = (0.97)^t$ and classify them as representing exponential growth or decay. (A2, M3)

i. Focus on exponential functions evaluated at integer inputs. (A1, M2)

CRITICAL AREA OF FOCUS #5, CONTINUED

Quadratic Functions and Modeling

Functions – Interpreting Functions

Analyze functions using different representations.

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* (A2, M3)

b. Focus on linear, quadratic, and exponential functions. (A1, M2)

BUILDING FUNCTIONS

Build a function that models a relationship between two quantities.

F.BF.1 Write a function that describes a relationship between two quantities. ★

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

i. Focus on linear and exponential functions. (A1, M1)

ii. Focus on situations that exhibit quadratic or exponential relationships. (A1, M2)

Build new functions from existing functions.

F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. M (A2, M3)

a. Focus on transformations of graphs of quadratic functions, except for $f(kx)$; (A1, M2)

LINEAR, QUADRATIC, AND EXPONENTIAL MODELS

Construct and compare linear, quadratic, and exponential models, and solve problems.

F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. ★ (A1, M2)